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Gravitational lensing to explore observational signatures of extra dimension of black holes in the braneworld scenario

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Abstract

Bending of light and gravitational lensing are two important observational tools to explore the nature of intervening massive object. *We see the braneworld scenario of our universe opens up the fascinating possibility of the existence of large extra spatial dimension. The braneworld black holes obey a modified mass-radius relationship compared to standard Schwarzschild black holes. Using the variational principle we calculate the bending angle of a light ray near the horizon of a braneworld black hole in the weak field limit. We next derive the expressions of several lensing quantities like the Einstein radius and the magnification for a point light source. The expressions are modified compared to the lensing quantities for standard Schwarzschild black holes and contain the scale of the extra dimensions.*

Keywords: *Branes, Gravitational Lensing, Branworld Black Hole, Einstein Radius*

Introduction

Generally the light rays observed by an observer far from any gravitational field should be seen to move in straight line. But the path of light will appear curved for an observer who is at rest relative to the earth, just as the path of light will appear curved for an observer who is in a uniformly accelerating rocket far from any gravitational field. So any acceleration can mimic a gravitational field. This is the principle of equivalence which leads us to believe that light rays will be bent by the gravitational field of a massive body. This can be tested by observing the apparent positions of stars during a solar eclipse. The stars are seen by light rays which just graze the surface of the Sun and the bending of these rays produces a distorted image of their positions. The deflection of light ray or bending of light can be calculated from Schwarzschild metric and this result was first tested during the eclipse in

1919 by Eddington (Gilmore & Tausch-Pebody, 2022). This result has been confirmed with radio interferometer methods with an uncertainty less than 1%. Bending of light is a direct proof of general theory of relativity. The Figure 1 shows how light ray bends in presence of any gravitational field.

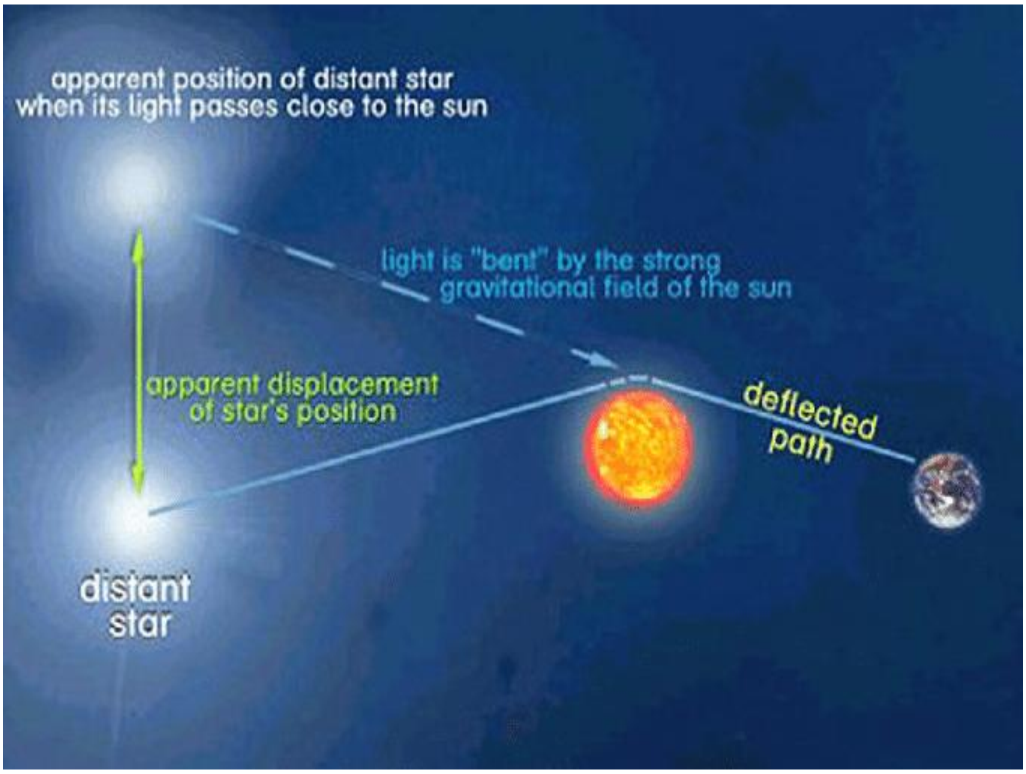

Figure 1. Light is bent by the strong gravitational field of the sun.
(http://light.physics.auth.gr/history/gtr/gtr_light_en.html).

Not only the light bends due to Sun's gravitational field, but beyond the solar system in the large scale there are also some massive objects which can block the direct view of sources. Since the intervening massive objects act as a lens to focus image of the distant sources to a new location, it is called gravitational lensing. It was first predicted in 1937 by Fritz Zwicky (Zwicky, 1937).In a system where lensing occurs the sources can be a quasar, the cosmic microwave background, galaxy and any lensing object which deflect the light by an amount can be anything with mass and energy. Not only for the visible light the lensing can occur, but this phenomenon may be occurred for any radiation. As a consequence of lensing, light rays bend from actual path. Suppose a light ray is coming from a distant quasar to us and if a massive body like black hole is blocking the direct view of it, the light will bend by the gravitational field of that black hole and we will see images of the distant quasar to new locations. The arena of gravitational lensing by braneworld black hole is a vast and potentially useful domain to establish the presence of signature of extra dimension by investigating some relevant observational quantities. A number of studies related to this topic have been carried out before (Maartens, 2004., Arkani-Hamed & Dimopolus, 1998., Majumdar & Mukherjee, 2005., Randall & Sundrum, 1999).

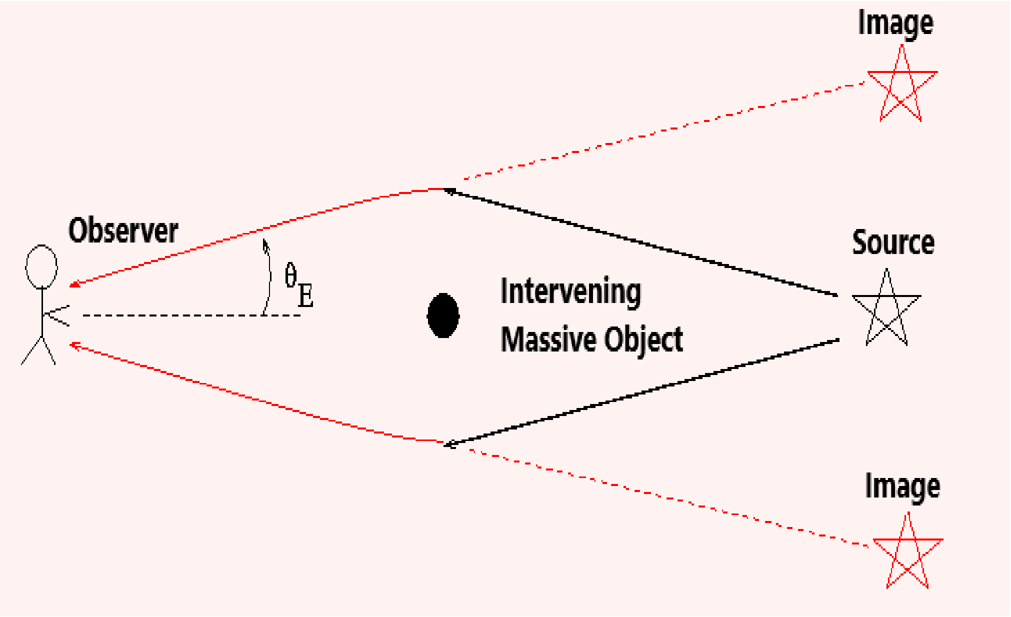

Figure 2. Gravitational lensing of a distant star by a massive object at the center is shown.

The structure of the paper is as follows. In Sec. 2, we briefly review about the Braneworld black hole. In Sec. 3, we study the gravitational lensing by Braneworld black hole and we show various types of interesting results. A summary of our paper and some concluding remarks are presented in Sec.4.

Braneworld Blackhole

In this paper we have taken the intervening massive object to be a black hole and it is a special type of black hole named “ braneworld black hole”. There is widespread activity in braneworld gravity in recent times. The braneworld scenario of our universe opens up the fascinating possibility of the existence of large extra spatial dimension(s) (Arkani-Hamed & Dimopolus, 1998). The hugely popular Randall-Sundrum (RS-II) braneworld model (Randall & Sundrum, 1999) is made consistent by the requirement that the standard model fields are confined to the brane, except for gravity which could also propagate into the bulk which may be of infinite extent but with curvature radius ‘l’. Current experiments probing the resultant modification of the Newtonian potential constrain the scale of the extra dimension to be'$1\leq 0.2$ mm'(Long *et al*., 2003) . A specific issue of interest in braneworld gravity is the formation and evolution of black holes (Kanti, 2004). Several types of black hole solutions have been obtained in the literature (Emparan *et al*., 2000, Dadhich *et al*., 2000).Black holes formed due to horizon sized density perturbations in the modified braneworld high energy phase of the early universe have a 5-dimensional Schwarzschild metric. The horizon size of such black holes is proportional to the square root of their mass, a feature that modifies the Hawking temperature, and consequently slows down the evaporation process (Majumdar, 2003).It has been shown that these black holes accrete radiation in the high energy phase which considerably prolongs their lifetimes. Some of them could survive up to the present era, thus acting as cold dark matter candidates. The braneworld high energy phase is rather conducive to the formation of primordial black hole binaries and gravitational waves from such coalescing binaries are likely to lie within the range of the next generation gravity wave detectors. On the other hand, super-horizon sized black holes which could be formed by various collapse mechanism possess different geometries, and might evaporate out rapidly as a consequence of Ads-CFT correspondence. Since the 5-dimensional fundamental scale could be several orders of magnitude below the Planck scale, a lot of current excitement stems from the possibility of braneworld black holes being produced in high energy particle collisions. Such a scenario would in principle, open up a direct experimental probe of extra dimensions and (5-d) Planck scale physics. This motivation has led to several specific proposals for black hole formation in TeV scale dynamics in colliders such as the LHC. Black hole production in cosmic ray showers has also been investigated with discussions on possible signatures.

The aim of this review paper is to investigate one potential avenue of observational signatures for extra dimensions. The non-trivial space time curvature around the vicinity of black holes could generate interesting motion for mass less and massive quanta passing near the horizon. Indeed, the bending of light around standard black holes leads to the resultant observable phenomenon of gravitational lensing which has been widely employed as a mechanism for detecting black holes in astrophysics. The analysis of light and particle motion in the context of braneworld gravity is complicated by the absence till date of unique analytical solutions for the metric representing compact objects in higher dimensions. The modification of the standard 1/r form of the Newtonian potential at small distances has nevertheless inspired several attempted solutions based on different physical requirements, and certain corresponding analyses have been performed on the trajectory of light rays and massive particles in such metrics. Some interesting results on orbits around rotating 5-dimensional black holes have been derived. In this paper we explore the lensing of optical sources by a braneworld black hole in the weak field limit. We first present the deflection angle for a light ray passing near the horizon of a braneworld black hole. The corresponding lensing quantities are presented next. Our analysis displays interesting departures of various lensing phenomena and quantities compared to the standard Schwarzschild metric.

Gravitational Lensing by Braneworld black hole

In our present work we consider a particular suggested geometry used in describing the space time metric near the horizon of a 5-dimensional braneworld black hole. Our analysis closely parallels the derivation of the angle of bending of light in the Schwarzschild metric using the variational principle for a null geodesic.

Small black holes formed with radius ‘r’ ≤ l in the early braneworld regime could grow in size by accreting radiation (Majumdar, Mehta &Luck, 2003., Randall & Sundrum, 2000). Even further growth to supermassive dimensions might be possible through accretion of the background “dark energy”. For such black holes the metric far away from the horizon is expected to be of the standard Schwarzschild form. Our interest is to focus on the region of a few horizon lengths surrounding the black hole where departures from the standard geometry could lead to interesting consequences, and where the deflection of light is accounted by the weak field limit to a good degree of approximation. The metric in this regionis given by (Majumdar, 2003)

$$dS^2 = -(1-r_{\hbar}{}^2/r^2)dt^2 + (1-r_{\hbar}{}^2/r^2)dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\Phi^2), \qquad (1)$$

where the horizon radius r(h) is related to the black hole mass M by

$$r_{\hbar} = 8/3\,\pi\ (l//l_4)\left(\frac{M}{M_4}\right) l_4{}^2 = PM, \qquad (2)$$

with ‘l’ representing the size of the extra dimension.l_4 andM_4 are denoting the 4-dimensional Planck length and mass respectively. Note that the Eq. (2) signifies the altered mass-radius relationship for a braneworld black hole (compared to mass-radius relationship for the standard Schwarzschild metric) and it is root of the all the new results for light deviation that follows from the braneworld metric.

We are interested in the motion of light ray using this space-time. Now the equation of motion is obtained in terms of the new variable u=1/r as

$$\frac{d^2u}{d\Phi^2} = u + 2P\,M\,u^3. \qquad (3)$$

And we notice that the above equation is different from standard Schwarzschild metric.

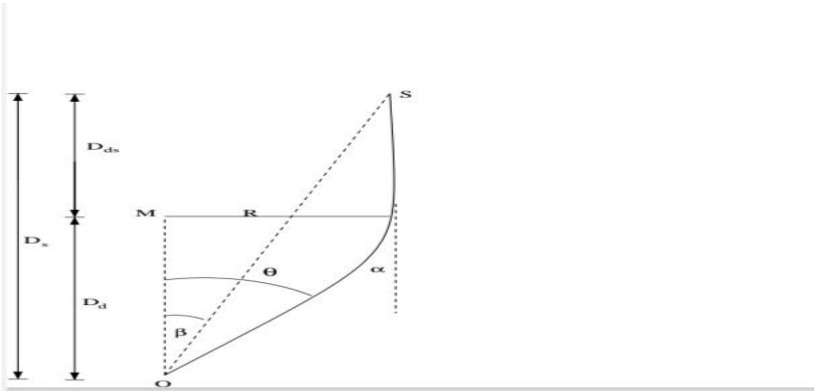

Figure 3. Gravitational lensing for point like mass object M. A light ray from the source ‘S’ which passes the lens at a distance ‘R’ is deflected by ‘α’. The observer sees an image of the source at angular position ‘θ’.

We obtain the total bending angle for this braneworld black hole space-time,

α= 3 (π/4) PM_4u^2. (4)

We have seen that the expression for the bending angle that we have derived contains the scale of extra dimension ‘l’. The expression for the bending angle of light ‘α’, we derived in the limit of a weak gravitational field follows essentially due to the modified mass-radius relation relationship for a braneworld black hole. The bending angle for the Schwarzschild metric which was derived in similar fashion (Randall & Sundrum, 2000) is given by

$\alpha_{Sch} = 4$ M u/$M^2{}_4$. (5)

The ratio of bending angle we obtained

α/α_{Sch}= 2lu, indicates that the deflection by a branworld black hole is more prominent for small impact parameter 1/u < l.

The deflection of light causes the gravitational lensing of a light a light source located behind the black hole for an observer. We have calculated various lensing quantities like the Einstein angle and the magnification. For point like sources, we derive these expressions in an approach that closely parallels the derivation of similar lensing quantities for the standard Schwarzschild metric (Schneider & Falco, 1992). From Figure 2, it is evident that the light rays reach the observer is governed by the equation, $\beta D_S = {}^{D_s}/_{D_d}$ R - D_{ds} α. (6)

Using the expression for the bending angle in Eq. (4), for our metric we have derived

β=θ - 3 (π/4)$\frac{D_{ds}}{Ds}\frac{PM}{\theta^2 {D_d}^2}$. (7)

Defining $\alpha_0{}^2$ =3 (π/4)$\frac{D_{ds}}{Ds}\frac{PM}{{D_d}^2}$ (8)

and$R_E = \alpha_0 D_d$, (9)

where α_0 and R_E are the dimensionless Einstein angle and the Einstein radius (Alock *et al.*, *1998*) respectively, we obtained Eq. (7) as

$\beta = \theta - \frac{{\alpha_0}^2}{\theta^2}$. (10)

This above Eq. (9) is known as lens equation which is obtained for braneworld metric.

The Schwarzschild lens equation which is given by,

$\beta = \theta - (\alpha_{Sc\hbar})^2/\theta$, (11)

in which the position θ can be obtained by solving Eq. (11), here the lens equation is quadratic in theta, and hence has two solutions, but the braneworld lens equation Eq.(10) is cubic in theta. It has only one real solution for the image position. A special case arises if the lens and the observer are linear i.e. $\beta = 0$. Then Eq. (10) becomes

$\theta^3 - {\alpha_0}^2 = 0$, (12)

whose real solution representing the Einstein ring (Schneider & Falco, 1992)and we obtain the value of θ as

$\theta = {\alpha_0}^{2/3}$. (13)

Another interesting quantity is the magnification of the image to be studied. The magnification μ is produced at the image position θ and we obtain this value in terms of the source position β by the Eq. (10). We compare the magnification produced in the Schwarzschild metric with our braneworld space-time and obtain that,

$\frac{\mu}{\mu_{Sc\hbar}} = (M_4/M)^{1/6} (\mathrm{l}//l_4)^{1/3} (l_4/D_d)^{1/6} (Ds/D_{ds})^{1/6}$. (14)

We have studied from the above equation that a larger size of the extra dimension would produce a brighter image. The magnification produced by such braneworld black hole lenses which could exist in our galactic halo, however, turns out to be diminished compared to the standard Schwarzschild black holes, except for extremely low masses. From Eq. (14), we obtain an interesting result $\frac{\mu}{\mu_{Sc\hbar}} \sim 10^{-4}$ using the value of D= $D_d(D_{ds}/Ds)$ ~ 10^{22}cm (relevant for the galactic halo objects) and $(M/M_4) \sim 10^{30}$. Black holes with such masses ($M/M_4 \sim 10^{30}$) have been classified as sub-lunar compact objects, and standard microlensing results (Alock *et al.*, *1998*) leave open the possibility of their existence in certain mass ranges as significant fractions of halo dark matter. From Eq. (14), we also notice, in astronomical lensing Schwarzschild black holes would produce brighter images than braneworld black holes for (M/M_4) >10^6. Thus our analysis shows that any branworld black holes present in the galactic halo would be harder to detect through weak field gravitational lensing where we have taken that distance of closest approach much greater than the Schwarzschild radius [13] (Raychaudhuri, A. K., Banerji, S ., and Banerjee, A. 1992).

Summary and Conclusions

Bending of light and gravitational lensing are two important observational tools to explore the nature of intervening massive objects. Braneworld black holes are the potential testing arenas of a rich theoretical structure associated with modified braneworld gravity and extra dimensions. A lot of present interest in the prospect of obtaining observable signatures of braneworld gravity is via the properties of the evaporation products of mini black holes produced either in particle collisions inside accelerators (Eardly, D. M., and Giddings, S. B. 2002**)**.. Another issue of interest is regarding primordial black which may be formed through the collapse overdense region in the braneworld high energy phase of the early universe. Braneworld black holes have entirely different metrics compared to four dimensional black holes and carry the signature of the extra dimension of their geometries. Currently a lot of ongoing works on finding observational signatures of intervening galaxies through gravitational lensing are carried out (Connor**.** L., & Ravi. V. 2022., Nazari. E. 2022., Turyshev. G Salva., and Toth. T. Viktor. 2022 ., Gao. X., Song, S., and Yang. J. 2019.)

The focus of this review has been to discuss the observational consequences of primordial braneworld black holes. In this review paper we have investigated weak gravitational lensing of a point-like optical source by abraneworld black hole. The study of particle and light motion in the geometry of higher dimensional black holes is a subject of recent interest, particularly so because of the proposed mechanisms of black hole formation in high energy particle collisions and cosmic ray showers. Further, as has been shown recently, braneworld black holes could survive as relics from the early universe and act as candidates of non-baryonic dark matter. It is feasible for primordial braneworld black holes to exist in the form of binaries (Majumdar, A. S., Mehta, A., and Luck, J. M. 2003) and gravitational waves from the coalescence of such binaries could be detected in the near future.

Thus, the exploration of the phenomenon of gravitational lensing by braneworld black holes could be of potential utility. The geometry of a braneworld black hole incorporates a different mass-radius relationship compared to a standard Schwarzschild black hole. In the above analysis, we have calculated the bending angle of light due to the gravitational potential of a braneworld black hole using the variational principle. The consistency of the derivation through the variational principle is confirmed using the general expression for deflection angle in a spherically symmetric metric. The expression for the bending angle that we have derived contains the scale of the extra dimension ‘l’. We have next explored the phenomenon of gravitational lensing in the weakfield limit. The expressions of lensing quantities like the Einstein angle and the magnification have been calculated in terms of the geometrical parameters and the size of the extra dimension. The differences of the seexpressions from the corresponding ones for Schwarzschild black hole lensing have been highlighted. Further interesting phenomena could be revealed through the analysis of strong gravitational lensing which in the context of the above braneworld geometry has been recently worked out (Eiroa. E. F., 2005.). Strong gravitational lensing in other braneworld metrics has also been studied (Whisker. R. 2005**.,** Majumdar. A. S., and Mukherjee. N., 2005., Majumdar. A. S., and Mukherjee. N., 2008.) Though our present observational capabilities might seem to restrict the status of such analyses to theoretical curiosities, further improvement in techniques might enable the fascinating possibility of discrimination of different gravity models through observable lensing effects in the near future. So in a nutshell the study of gravitational lensing by black holes in the braneworld scenario could be very useful and important in the context of searching possible observational signatures of these objects to unfold several other interesting features of our universe.

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